

Introduction of the Kifilideen’s Extermination and Determinant of Matrix (KEDM) Method for Resolving Multivariable Linear Systems with Two, Three and Four Unknowns

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Abstract

The Gaussian extermination technique poses significant challenges when applied to systems of linear equations with three or four unknowns, as it involves multiple steps and requires a high number of arithmetic operations, making the method less efficient and more complex to implement and understand. Furthermore, the Jacobian and Gauss-Seidel methods produce approximate solutions that may not accurately represent the true solution. Additionally, LU decomposition method can introduce round – off errors, leading to inaccurate solutions. The graphical method is also impractical for systems with more than two unknowns, as visualization and interpretation become increasingly difficult. Cramer’s rule, used for resolving large systems of linear equations, is computationally complex and inefficient due to complexity of determinant calculations. Therefore, a simpler and more efficient technique is needed for resolving linear systems of simultaneous equations with two, three, and four unknowns. This study introduces Kifilideen’s Extermination and Determinant of Matrix (KEDM) Method for resolving multivariable linear systems with two, three, and four unknowns. The KEDM method employs a progressive extermination technique to narrow down the number of unknowns of a system of simultaneous equations using a determinant of matrix layout. This method was developed to efficiently determine the solution of linear systems of simultaneous equations. The KEDM method was tested on linear systems of simultaneous equations with two, three and four unknowns to evaluate its effectiveness and simplicity. The results show that the KEDM method involves only 2×2 determinant of matrix calculations, making it simpler, easier, more intuitive, less computationally expensive and more efficient to implement and understand.

Keywords: LU Decomposition, Kifilideen Method, Jacobian Method, Gauss-Seidel Method, Extermination Method, Graphical Method

Introduction

A multivariable linear system of simultaneous equations is a system of equations where each equation contains the same set of unknown values of a set of unknown variables, each with a degree of one (Samuel, 2012). In each equation, variables have coefficients that sum up to give an output. For instance, Chen et al. (2013) and Luo et al. (2021) provided a typical example of a multivariable linear system of simultaneous equations:

$$\begin{aligned} \alpha i_1 + \beta i_2 + \dots + \gamma i_n &= u \\ \mu i_1 + \lambda i_2 + \dots + \theta i_n &= v \\ &\vdots \\ \rho i_1 + \sigma i_2 + \dots + \tau i_n &= w \end{aligned}$$

Where i_1, i_2, \dots , and i_n are the unknown variables; $\alpha, \beta, \dots, \gamma, \mu, \lambda, \dots, \theta, \rho, \sigma, \dots, \tau$ are the coefficients of the variables and u, v, \dots, w are the outputs of the system of the linear equations. The number of unknown variables determines the required number of equations needed to resolve for each variable. Multivariable linear system of simultaneous equations arise in various fields, including biology, medicine, agriculture, science, engineering, computer science, finance, economic growth, population dynamics, Kifilideen's Arithmetic Matrix Progression Sequence of infinite and finite terms, Kifilideen's Trinomial Theorem of positive and negative power of n , data analysis, electrical circuit analysis, environmental science, navigation and transportation, investment portfolio, traffic flow, climate modeling, genetics, pharmacokinetics, machine learning, water quality modeling and structural analysis and linear programming (Park, 2005; Chenini and Khemiri, 2009; Huang and Xie, 2013; Adu, 2014, Osanyinpeju, 2020a; Osanyinpeju, 2023; Sandoval, 2024).

Formulation of systems of linear equations in supply and demand can be utilized to determine the optimal production of different products produced in a company. More so, the system of linear equations is generated by a business investor to determine the optimal investment portfolio in assets like stocks, bonds and real estate. In analysis of electrical circuits, a system of linear equations is generated to determine the value of the current and resistance at different parts of the electrical circuit. Furthermore, for structural analysis, a system of linear equations is produced to

determine the forces and moment acting on the columns and beams of a structural building. More so, in the agriculture system, farmers could generate a system of linear equations to determine the optimal allocation of resources such as water and fertilizer available so as to maximize crop yield.

In analyzing real world problems of Kifilideen's Arithmetic Matrix Progression Sequence of finite and infinite terms, system of linear equations of three variables are generated where the variables are migration level value, k , migration step value, i , and the first term, f of the properties of the product under study (Osanyinpeju, 2024). To determine the value of the three variables k, i and f , system of three linear equations are required. More so, in the analysis of the Kifilideen's Trinomial Theorem of positive and negative power of n to determine the power combination of a term in the expansion of Kifilideen's Trinomial Theorem, system of linear equations is generated which contain three variables k, i and f . Where k, i and f are the first, second and third component of the power combination respectively (Osanyinpeju, 2020b; Osanyinpeju, 2022).

Advances in mathematics have yielded numerous groundbreaking discoveries, notably the development of innovative mathematical methods for resolving complex problems (Beutelspacher, 2018; Yadav, 2019). To remain current in the field, regular and application of mathematical principles are essential for mastering new concepts (Osanyinpeju, 2022). Effective mastery of novel mathematical methods necessitates consistent application and practice. Repeated engagement with these concepts fosters deeper comprehension, whereas limited exposure can impede knowledge retention (Osanyinpeju, 2019; Osanyinpeju, 2020c).

Several approaches have been developed to resolve linear systems of simultaneous equations, including numerical and algebraic techniques. Traditional solution methods include extermination, substitution, graphical, Cramer's rule, inverse matrix, Gaussian extermination, Jacobian iteration, Gauss-Seidel, and LU decomposition (Ugboduma, 2012; Woollard, 2015). Gaussian extermination specifically transforms simultaneous equations into triangular form by eliminating select elements (Grcar, 2011).

However, existing methods have limitations. Gaussian extermination involves multiple steps, including forward extermination, back substitution, and pivoting, which can be computationally complex (Gharib et al., 2015). Cramer's rule of resolving simultaneous equations for four

variables system involves 4×4 , 3×3 and 2×2 determinant of matrices calculations and for the three variables system it involves 3×3 and 2×2 determinants of matrices calculations. Cramer's rule, used for resolving large systems of linear equations, is computationally complex and inefficient due to complexity of determinant calculations (Babarinsa and Kamarulhaili, 2017). For a large system, Cramer's rule requires calculating multiple complex determinants, which can be time consuming and prone to errors (Luo et al., 2021).

Jacobian and Gauss – Seidel Methods involve iterative calculations and initial guesses, requiring convergence tests to ensure accuracy (Kaur and Kaur, 2012). The convergence to such a solution requires multiple iterations and depends on a good initial guess. Iterative methods are sensitive to the conditioning of the coefficient matrix and diagonal elements to be non-zero and dominant (Gharib et al., 2015). Graphical methods are limited to two-variable systems, as higher-variable systems become challenging to visualize and interpret. To address these limitations, this study introduces Kifilideen's Extermination and Determinant of Matrix (KEDM) Method for resolving multivariable linear systems with two, three, and four unknowns.

Material and methods

The Kifilideen's Extermination and Determinant of Matrix (KEDM) Method is a systematic approach for resolving linear systems of equations. This method employs a determinant of matrix formulation to progressively eliminate variables, thereby reducing the number of unknowns. The KEDM method is specifically designed to efficiently resolve multivariable linear systems with two, three, and four unknowns. The method involves transforming the system's coefficients into a series of 2×2 determinants of matrices. Each KEDM method layout comprises the arrangement of coefficients within a determinant of matrix, where the variable to be determined is multiplied by this determinant to produce an output equation. This output equation, along with the coefficients, is presented in a determinant of matrix layout.

Kifilideen's Extermination and Determinant of Matrix (KEDM) Method to Resolve Linear System of Two Unknowns

This section outlines the step-by-step procedure for developing the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method for resolving linear systems with two unknowns. Consider a system of two linear equations with two unknowns, denoted as:

$$\alpha i_1 + \beta i_2 = v \tag{1}$$

$$\mu i_1 + \theta i_2 = w \quad (2)$$

The KEDM method can be generated using the following steps:

Step 1: Extermination of unknown i_2

Apply the extermination technique to reduce the number of unknowns by first exterminating the unknown i_2 from equations (1) and (2) within the determinant of the matrix layout.

$$\begin{vmatrix} \alpha & \beta \\ \mu & \theta \end{vmatrix} i_1 + \begin{vmatrix} \beta & \beta \\ \theta & \theta \end{vmatrix} i_2 = \begin{vmatrix} v & \beta \\ w & \theta \end{vmatrix}$$

Step 2: Establish the KEDM method to resolve i_1

$$\begin{matrix} C_{i_1} & C_{i_2} & C_o & C_{i_2} \\ \begin{vmatrix} \alpha & \beta \\ \mu & \theta \end{vmatrix} i_1 = \begin{vmatrix} v & \beta \\ w & \theta \end{vmatrix} \end{matrix} \quad (3)$$

The KEDM methods provide a solution for i_1 in a system of linear equations with two unknowns, i_1 and i_2 , as expressed in equation (3).

Step 3: Extermination of unknown i_1

Apply the extermination technique to reduce the number of unknowns by first exterminating the unknown i_1 from equations (1) and (2) within the determinant of the matrix layout.

$$\begin{vmatrix} \alpha & \alpha \\ \mu & \mu \end{vmatrix} i_1 + \begin{vmatrix} \beta & \alpha \\ \theta & \mu \end{vmatrix} i_2 = \begin{vmatrix} v & \alpha \\ w & \mu \end{vmatrix}$$

Step 4: Establish the KEDM method to resolve i_2

$$\begin{matrix} C_{i_2} & C_{i_1} & C_o & C_{i_1} \\ \begin{vmatrix} \beta & \alpha \\ \theta & \mu \end{vmatrix} i_2 = \begin{vmatrix} v & \alpha \\ w & \mu \end{vmatrix} \end{matrix} \quad (4)$$

The KEDM methods provide a solution for i_2 in a system of linear equations with two unknowns, i_1 and i_2 , as expressed in equation (4).

KEDM method structure

The coefficients of the unknowns in (1) and (2) are assigned as follows:

- Unknown i_1 : coefficients: ' α ' and ' μ '
- Unknown i_2 : coefficients: ' β ' and ' θ '
- System's output constant: ' v ' and ' w '

KEDM method dimensions

The KEDM method equation system has four columns and two rows for linear system of two unknowns. To find unknown i_1 or i_2 , the columns represent coefficients of unknowns i_1 or i_2 , system's output constants, and coefficients of unknown i_1 or i_2 .

Column configuration

To find unknown i_1 or i_2 , the columns in the KEDM method are configured as follows:

- Column 1: coefficients of the unknown being resolved for.
- Columns 2 and 4: coefficients of the first exterminated unknown
- Column 3: system's output constants

Row configuration

The rows in the KEDM method for two unknowns are configured as follows:

- Row 1: corresponds to the coefficients of equation (1)
- Row 2: corresponds to the coefficients of equation (2)

Kifilideen's Extermination and Determinant of Matrix (KEDM) Method to Resolve Linear System of Three Unknowns

This section outlines the step-by-step procedure for developing the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method for resolving linear systems with three unknowns.

Consider a system of three linear equations with three unknowns, denoted as:

$$\alpha i_1 + \beta i_2 + \gamma i_3 = u \tag{5}$$

$$\mu i_1 + \theta i_2 + \varphi i_3 = v \tag{6}$$

$$\chi i_1 + \Omega i_2 + \rho i_3 = w \tag{7}$$

The KEDM method can be generated using the following steps:

Step 1: Extermination of unknown i_2

To construct the KEDM method for determining unknown i_1 , utilize a systematic reduction approach. Initially, exterminate one of the unknowns, either i_2 or i_3 . Subsequently, the remaining unknown is removed within the determinant of the matrix framework. In the given equations, we first eliminate i_2 using equations (5) and (6), and then again using equations (6) and (7). The resulting expressions are:

From equations (5) and (6), we obtain:

$$\begin{aligned} \begin{vmatrix} \alpha & \beta \\ \mu & \theta \end{vmatrix} i_1 + \begin{vmatrix} \beta & \beta \\ \theta & \theta \end{vmatrix} i_2 + \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \end{vmatrix} i_3 &= \begin{vmatrix} u & \beta \\ v & \theta \end{vmatrix} \\ \begin{vmatrix} \alpha & \beta \\ \mu & \theta \end{vmatrix} i_1 + \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \end{vmatrix} i_3 &= \begin{vmatrix} u & \beta \\ v & \theta \end{vmatrix} \end{aligned} \quad (8)$$

From equations (6) and (7), we obtain:

$$\begin{aligned} \begin{vmatrix} \mu & \theta \\ \lambda & \Omega \end{vmatrix} i_1 + \begin{vmatrix} \theta & \theta \\ \Omega & \Omega \end{vmatrix} i_2 + \begin{vmatrix} \varphi & \theta \\ \rho & \Omega \end{vmatrix} i_3 &= \begin{vmatrix} v & \theta \\ w & \Omega \end{vmatrix} \\ \begin{vmatrix} \mu & \theta \\ \lambda & \Omega \end{vmatrix} i_1 + \begin{vmatrix} \varphi & \theta \\ \rho & \Omega \end{vmatrix} i_3 &= \begin{vmatrix} v & \theta \\ w & \Omega \end{vmatrix} \end{aligned} \quad (9)$$

Step 2: Extermination of unknown i_3

Subsequently, equations (8) and (9) are utilized to further exterminate i_3 , ultimately yielding the KEDM method to determine i_1

Step 4: Establish the KEDM method to resolve i_1

$$\begin{aligned} C_{i_1} \quad C_{i_2} \quad C_{i_3} \quad C_{i_2} \quad C_o \quad C_{i_2} \quad C_{i_3} \quad C_{i_2} \\ \begin{vmatrix} \alpha & \beta \\ \mu & \theta \\ \mu & \theta \\ \lambda & \Omega \end{vmatrix} \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \\ \varphi & \theta \\ \rho & \Omega \end{vmatrix} i_1 = \begin{vmatrix} u & \beta \\ v & \theta \\ v & \theta \\ w & \Omega \end{vmatrix} \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \\ \varphi & \theta \\ \rho & \Omega \end{vmatrix} \end{aligned} \quad (10)$$

The KEDM methods provide a solution for i_1 in a system of linear equations with three unknowns, i_1 , i_2 and i_3 , as expressed in equation (10).

Step 5: Extermination of unknown i_1

Subsequently, equations (8) and (9) are utilized to further exterminate i_1 , ultimately yielding the KEDM method to determine i_3

Step 6: Establish the KEDM method to resolve i_3

$$\begin{aligned} C_{i_3} \quad C_{i_2} \quad C_{i_1} \quad C_{i_2} \quad C_o \quad C_{i_2} \quad C_{i_1} \quad C_{i_2} \\ \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \\ \varphi & \theta \\ \rho & \Omega \end{vmatrix} \begin{vmatrix} \alpha & \beta \\ \mu & \theta \\ \mu & \theta \\ \lambda & \Omega \end{vmatrix} i_3 = \begin{vmatrix} u & \beta \\ v & \theta \\ v & \theta \\ w & \Omega \end{vmatrix} \begin{vmatrix} \alpha & \beta \\ \mu & \theta \\ \mu & \theta \\ \lambda & \Omega \end{vmatrix} \end{aligned} \quad (11)$$

Step 7: KEDM method for resolving unknown i_2

Building on the structural framework introduced in equations (10) and (11), we can derive the KEDM method for resolving unknown i_2 . This specific KEDM method configuration to resolve unknown i_2 involves the sequential extermination of unknowns i_1 and i_3 , yielding the equation (12).

$$\begin{matrix} C_{i_2} & C_{i_1} & C_{i_3} & C_{i_1} & C_o & C_{i_1} & C_{i_3} & C_{i_1} \\ \left| \begin{matrix} \beta \\ \theta \\ \theta \\ \Omega \end{matrix} \right| & \left| \begin{matrix} \alpha \\ \mu \\ \mu \\ \lambda \end{matrix} \right| & \left| \begin{matrix} \gamma \\ \varphi \\ \varphi \\ \rho \end{matrix} \right| & \left| \begin{matrix} \alpha_1 \\ \mu \\ \mu \\ \lambda \end{matrix} \right| & = & \left| \begin{matrix} u \\ v \\ v \\ w \end{matrix} \right| & \left| \begin{matrix} \alpha \\ \mu \\ \mu \\ \lambda \end{matrix} \right| & \left| \begin{matrix} \gamma \\ \varphi \\ \varphi \\ \rho \end{matrix} \right| & \left| \begin{matrix} \alpha_1 \\ \mu \\ \mu \\ \lambda \end{matrix} \right| \end{matrix} \quad (12)$$

KEDM method structure

The coefficients of the unknowns in (5), (6) and (7) are assigned as follows:

- Unknown i_1 : coefficients: ' α' , ' μ' and ' λ' '
- Unknown i_2 : coefficients: ' β' , ' θ' and Ω
- Unknown i_3 : coefficients: ' γ' , ' φ' and ' ρ'
- System's output constant: ' u' , ' v' and ' w'

KEDM method dimensions

The KEDM method equation system has eight columns and four rows for linear system of three unknowns.

Column configuration

To find unknown i_1 , i_2 or i_3 the columns in the KEDM method are configured as follows:

- Column 1: the coefficient of the unknown being resolved for.
- Column 2, 4, 6 and 8: the coefficients of the first exterminated unknown
- Column 3 and 7: the coefficients of the second exterminated unknown
- Column 5: system's output constants

Row configuration

The rows in the KEDM method for three unknowns are configured as follows:

- Row 1: corresponds to the coefficients of equation (5)
- Row 2: corresponds to the coefficients of equation (6)
- Row 3: corresponds to the coefficients of equation (6)
- Row 4: corresponds to the coefficients of equations (7)

Kifilideen's Extermination and Determinant of Matrix (KEDM) Method to Resolve Linear System of Four Unknowns

This section outlines the step-by-step procedure for developing the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method for resolving linear systems with four unknowns.

Consider a system of four linear equations with four unknowns, denoted as:

$$\alpha i_1 + \beta i_2 + \gamma i_3 + \delta i_4 = t \quad (13)$$

$$\mu i_1 + \theta i_2 + \varphi i_3 + \tau i_4 = u \quad (14)$$

$$\lambda i_1 + \Omega i_2 + \rho i_3 + \delta i_4 = v \quad (15)$$

$$\Psi i_1 + \eta i_2 + \sigma i_3 + \emptyset i_4 = w \quad (16)$$

The KEDM method can be generated using the following steps:

Step 1: Extermination of unknown i_2

To construct the KEDM method for determining unknown i_1 , employ a systematic reduction approach. This involves exterminating unknowns in a specific sequence to isolate i_1 . Begin by exterminating one unknown (i_2 , i_3 or i_4) from the given equations. Then, remove the remaining unknown within the determinant of the matrix framework. In this instance, we sequentially exterminate i_2 using the following equation pairs:

- Equations (13) and (14)
- Equations (14) and (15)
- Equations (15) and (16)

The resulting expressions are:

From equations (13) and (14):

$$\begin{vmatrix} \alpha & \beta \\ \mu & \theta \end{vmatrix} i_1 + \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \end{vmatrix} i_3 + \begin{vmatrix} \delta & \beta \\ \tau & \theta \end{vmatrix} i_4 = \begin{vmatrix} t & \beta \\ u & \theta \end{vmatrix} \quad (17)$$

From equations (14) and (15):

$$\begin{vmatrix} \mu & \theta \\ \lambda & \Omega \end{vmatrix} i_1 + \begin{vmatrix} \varphi & \theta \\ \rho & \Omega \end{vmatrix} i_3 + \begin{vmatrix} \tau & \theta \\ \delta & \Omega \end{vmatrix} i_4 = \begin{vmatrix} u & \theta \\ v & \Omega \end{vmatrix} \quad (18)$$

From equations (15) and (16):

$$\begin{vmatrix} \lambda & \Omega \\ \Psi & \eta \end{vmatrix} i_1 + \begin{vmatrix} \rho & \Omega \\ \sigma & \eta \end{vmatrix} i_3 + \begin{vmatrix} \delta & \Omega \\ \emptyset & \eta \end{vmatrix} i_4 = \begin{vmatrix} v & \Omega \\ w & \eta \end{vmatrix} \quad (19)$$

Step 2: Extermination of unknown i_3

Subsequently, we exterminate i_3 in the second step using equations (17) and (18), as well as equations (18) and (19). The resulting expressions are:

Derived from equations (17) and (18):

$$\begin{vmatrix} \alpha & \beta \\ \mu & \theta \\ \mu & \theta \\ \lambda & \Omega \end{vmatrix} \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \\ \varphi & \theta \\ \rho & \Omega \end{vmatrix} i_1 + \begin{vmatrix} \delta & \beta \\ \tau & \theta \\ \tau & \theta \\ \delta & \Omega \end{vmatrix} \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \\ \varphi & \theta \\ \rho & \Omega \end{vmatrix} i_4 = \begin{vmatrix} t & \beta \\ u & \theta \\ u & \theta \\ v & \Omega \end{vmatrix} \begin{vmatrix} \gamma & \beta \\ \varphi & \theta \\ \varphi & \theta \\ \rho & \Omega \end{vmatrix} \quad (20)$$

Derived from equations (18) and (19):

$$\begin{pmatrix} \mu & \theta \\ \lambda & \Omega \\ \psi & \eta \end{pmatrix} \begin{pmatrix} \varphi & \theta \\ \rho & \Omega \\ \sigma & \eta \end{pmatrix} i_1 + \begin{pmatrix} \tau & \theta \\ \delta & \Omega \\ \emptyset & \eta \end{pmatrix} \begin{pmatrix} \varphi & \theta \\ \rho & \Omega \\ \sigma & \eta \end{pmatrix} i_4 = \begin{pmatrix} u & \theta \\ v & \Omega \\ w & \eta \end{pmatrix} \begin{pmatrix} \varphi & \theta \\ \rho & \Omega \\ \sigma & \eta \end{pmatrix} \quad (21)$$

Step 3: Extermination of unknown i_4

Subsequently, equations (20) and (21) are utilized to further exterminate i_4 , ultimately yielding the KEDM method to determine i_1

Step 4: Establish the KEDM method to resolve i_1

$$\begin{pmatrix} C_{i_1} & C_{i_2} & C_{i_3} & C_{i_2} & C_{i_4} & C_{i_2} & C_{i_3} & C_{i_2} \\ \alpha & \beta & \gamma & \beta & \delta & \beta & \gamma & \beta \\ \mu & \theta & \varphi & \theta & \tau & \theta & \varphi & \theta \\ \mu & \theta & \varphi & \theta & \tau & \theta & \varphi & \theta \\ \lambda & \Omega & \rho & \Omega & \delta & \Omega & \rho & \Omega \\ \mu & \theta & \varphi & \theta & \tau & \theta & \varphi & \theta \\ \lambda & \Omega & \rho & \Omega & \delta & \Omega & \rho & \Omega \\ \lambda & \Omega & \rho & \Omega & \delta & \Omega & \rho & \Omega \\ \psi & \eta & \sigma & \eta & \emptyset & \eta & \sigma & \eta \end{pmatrix} i_1 = \begin{pmatrix} C_o & C_{i_2} & C_{i_3} & C_{i_2} & C_{i_4} & C_{i_2} & C_{i_3} & C_{i_2} \\ t & \beta & \gamma & \beta & \delta & \beta & \gamma & \beta \\ u & \theta & \varphi & \theta & \tau & \theta & \varphi & \theta \\ u & \theta & \varphi & \theta & \tau & \theta & \varphi & \theta \\ v & \Omega & \rho & \Omega & \delta & \Omega & \rho & \Omega \\ u & \theta & \varphi & \theta & \tau & \theta & \varphi & \theta \\ v & \Omega & \rho & \Omega & \delta & \Omega & \rho & \Omega \\ v & \Omega & \rho & \Omega & \delta & \Omega & \rho & \Omega \\ w & \eta & \sigma & \eta & \emptyset & \eta & \sigma & \eta \end{pmatrix} \quad (22)$$

The KEDM methods provide a solution for i_1 in a system of linear equations with four unknowns, i_1, i_2, i_3 , and i_4 as expressed in equation (22).

Step 5: Extermination of unknown i_1

Subsequently, equations (20) and (21) are utilized to further exterminate i_1 , ultimately yielding the KEDM method to determine i_4

Step 6: Establish the KEDM method to resolve i_4

$$\begin{pmatrix} C_{i_4} & C_{i_2} & C_{i_3} & C_{i_2} & C_{i_1} & C_{i_2} & C_{i_3} & C_{i_2} \\ \delta & \beta & \gamma & \beta & \alpha & \beta & \gamma & \beta \\ \tau & \theta & \varphi & \theta & \mu & \theta & \varphi & \theta \\ \tau & \theta & \varphi & \theta & \mu & \theta & \varphi & \theta \\ \delta & \Omega & \rho & \Omega & \lambda & \Omega & \rho & \Omega \\ \tau & \theta & \varphi & \theta & \mu & \theta & \varphi & \theta \\ \delta & \Omega & \rho & \Omega & \lambda & \Omega & \rho & \Omega \\ \delta & \Omega & \rho & \Omega & \lambda & \Omega & \rho & \Omega \\ \emptyset & \eta & \sigma & \eta & \psi & \eta & \sigma & \eta \end{pmatrix} i_4 = \begin{pmatrix} C_o & C_{i_2} & C_{i_3} & C_{i_2} & C_{i_1} & C_{i_2} & C_{i_3} & C_{i_2} \\ t & \beta & \gamma & \beta & \alpha & \beta & \gamma & \beta \\ u & \theta & \varphi & \theta & \mu & \theta & \varphi & \theta \\ u & \theta & \varphi & \theta & \mu & \theta & \varphi & \theta \\ v & \Omega & \rho & \Omega & \lambda & \Omega & \rho & \Omega \\ u & \theta & \varphi & \theta & \mu & \theta & \varphi & \theta \\ v & \Omega & \rho & \Omega & \lambda & \Omega & \rho & \Omega \\ v & \Omega & \rho & \Omega & \lambda & \Omega & \rho & \Omega \\ w & \eta & \sigma & \eta & \psi & \eta & \sigma & \eta \end{pmatrix} \quad (23)$$

Step 7: KEDM method for resolving unknown i_2

Building on the structural framework introduced in equations (22) and (23), we can derive the KEDM method for resolving unknown i_2 . This specific KEDM method configuration to resolve

unknown i_2 involves the sequential extermination of unknowns i_1 , i_3 and i_4 yielding the equation (24).

$$\begin{array}{cccc|cccc|cccc|cccc}
 C_{i_2} & C_{i_1} & C_{i_3} & C_{i_1} & C_{i_4} & C_{i_1} & C_{i_3} & C_{i_1} & C_o & C_{i_1} & C_{i_3} & C_{i_1} & C_{i_4} & C_{i_1} & C_{i_3} & C_{i_1} \\
 \left\| \begin{array}{c} \beta \\ \theta \\ \theta \\ \Omega \\ \theta \\ \Omega \\ \eta \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \gamma \\ \varphi \\ \varphi \\ \rho \\ \varphi \\ \rho \\ \sigma \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \delta \\ \tau \\ \tau \\ \delta \\ \tau \\ \delta \\ \emptyset \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \gamma \\ \varphi \\ \varphi \\ \rho \\ \varphi \\ \rho \\ \sigma \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} t \\ u \\ u \\ v \\ u \\ v \\ w \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \gamma \\ \varphi \\ \varphi \\ \rho \\ \varphi \\ \rho \\ \sigma \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \delta \\ \tau \\ \tau \\ \delta \\ \tau \\ \delta \\ \emptyset \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \gamma \\ \varphi \\ \varphi \\ \rho \\ \varphi \\ \rho \\ \sigma \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\|
 \end{array} i_2 = \quad (24)$$

Step 8: KEDM method for resolving unknown i_3

Building on the structural framework introduced in equations (22) and (23), we can derive the KEDM method for resolving unknown i_3 . This specific KEDM method configuration to resolve unknown i_3 involves the sequential extermination of unknowns i_1 , i_2 and i_4 yielding the equation (25).

$$\begin{array}{cccc|cccc|cccc|cccc}
 C_{i_3} & C_{i_1} & C_{i_2} & C_{i_1} & C_{i_4} & C_{i_1} & C_{i_2} & C_{i_1} & C_o & C_{i_1} & C_{i_2} & C_{i_1} & C_{i_4} & C_{i_1} & C_{i_2} & C_{i_1} \\
 \left\| \begin{array}{c} \gamma \\ \varphi \\ \varphi \\ \rho \\ \varphi \\ \rho \\ \rho \\ \sigma \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \beta \\ \theta \\ \theta \\ \Omega \\ \theta \\ \Omega \\ \eta \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \delta \\ \tau \\ \tau \\ \delta \\ \tau \\ \delta \\ \emptyset \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \beta \\ \theta \\ \theta \\ \Omega \\ \theta \\ \Omega \\ \eta \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} t \\ u \\ u \\ v \\ u \\ v \\ w \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \beta \\ \theta \\ \theta \\ \Omega \\ \theta \\ \Omega \\ \eta \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \delta \\ \tau \\ \tau \\ \delta \\ \tau \\ \delta \\ \emptyset \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\| & \left\| \begin{array}{c} \beta \\ \theta \\ \theta \\ \Omega \\ \theta \\ \Omega \\ \eta \end{array} \right\| & \left\| \begin{array}{c} \alpha \\ \mu \\ \mu \\ \lambda \\ \mu \\ \lambda \\ \Psi \end{array} \right\|
 \end{array} i_3 = \quad (25)$$

KEDM method structure

The coefficients of the unknowns in (13), (14), (15) and (16) are assigned as follows:

- Unknown i_1 : coefficients: ' α ', ' μ ', ' λ ' and ' Ψ '
- Unknown i_2 : coefficients: ' β ', ' θ ', ' Ω ' and ' η '
- Unknown i_3 : coefficients: ' γ ', ' φ ', ' ρ ' and ' σ '
- Unknown i_4 : coefficients: ' δ ', ' τ ', ' δ ' and ' \emptyset '
- System's output constant: ' t ', ' u ', ' v ' and ' w '

KEDM method dimensions

The KEDM method equation system has sixteen columns and eight rows for linear system of four unknowns.

Column configuration

To find unknown i_1 , i_2 , i_3 or i_4 the columns in the KEDM method are configured as follows:

- Column 1: the coefficient of the unknown being resolved for.
- Column 2, 4, 6, 8, 10, 12, 14 and 16: the coefficients of the first exterminated unknown
- Column 3, 7, 11 and 15: the coefficients of the second exterminated unknown
- Column 5 and 13: the coefficients of the third exterminated unknown
- Column 9: system's output constants

Row configuration

The rows in the KEDM method for four unknowns are configured as follows:

- Row 1: corresponds to the coefficients of equation (13)
- Row 2: corresponds to the coefficients of equation (14)
- Row 3: corresponds to the coefficients of equation (14)
- Row 4: corresponds to the coefficients of equations (15)
- Row 5: corresponds to the coefficients of equation (14)
- Row 6: corresponds to the coefficients of equation (15)
- Row 7: corresponds to the coefficients of equation (15)
- Row 8: corresponds to the coefficients of equations (16)

Results and discussion

This study employed the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method to resolve multivariable linear systems with two, three, and four unknowns. The primary objective was to assess the method's effectiveness and simplicity.

Application of the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method to Resolve Linear System of Two unknowns

Solve the following linear system with two unknowns using the KEDM method.

$$\begin{aligned}4R_1 - 7R_2 &= -50 \\ -6R_1 + 9R_2 &= 60\end{aligned}$$

Solution

Utilizing the KEDM method to determine the value of R_1 , with R_2 exterminated first, yields the following result:

$$\begin{array}{cc|c} C_{R_1} & C_{R_2} & C_o \\ \hline 4 & -7 & -50 \\ -6 & 9 & -7 \end{array} R_1 = \begin{array}{cc|c} -50 & -7 & \\ \hline 60 & 9 & \end{array} \quad (26)$$

$$\begin{aligned} -6R_1 &= -30 \\ R_1 &= \frac{-30}{-6} = 5 \end{aligned} \quad (27)$$

Utilizing the KEDM method to determine the value of R_2 , with R_1 exterminated first, yields the following result:

$$\begin{array}{cc|c} C_{R_2} & C_{R_1} & C_o \\ \hline -7 & 4 & -50 \\ 9 & -6 & 4 \end{array} R_2 = \begin{array}{cc|c} -50 & 4 & \\ \hline 60 & -6 & \end{array} \quad (28)$$

$$\begin{aligned} 6R_2 &= 60 \\ R_2 &= \frac{60}{6} = 10 \end{aligned} \quad (29)$$

$$R_1 = 5, \text{ and } R_2 = 10 \quad (30)$$

Various methods exist for resolving linear systems with two unknowns. The Kifilideen's Extermination and Determinant of Matrix (KEDM) Method provides a systematic approach to solving linear systems of equations with two unknowns, as demonstrated in equations (26) to (30). This approach ensures consistency and reliability in both procedure and usage.

In contrast, Cramer's rule offers an alternative approach to resolving linear systems with two unknowns. It employs a different methodology and matrix arrangement; it ultimately relies on determinants of matrices. Both Cramer's rule and KEDM method are effective and straightforward technique for resolving linear systems with two unknowns.

Another approach is the inverse matrix method, which entails a multi-stage process. This process includes calculating cofactors, obtaining the transpose of the cofactors, determining the determinant, and finding the inverse of the matrix. Consequently, this procedure can be perceived as intricate, cumbersome, and prone to errors. The procedure for using the inverse matrix method is complex, nuanced, and potentially challenging to follow.

The graphical method is also an option, although it has significant limitations. Notably, it lacks accuracy and relies on general approximations, particularly when dealing with fractional or

decimal solutions. Furthermore, if the point of intersection represents a fraction or decimal value, accurately determining the solution from the graph becomes challenging. Consequently, the graphical method is best suited for solving simple linear systems with two unknowns, where solutions are integers or easy-to-approximate values.

According to Ghosh (2022), the Gaussian extermination method for resolving linear systems with two unknowns is a time-consuming process. It involves extensive mathematical computations, making it a slow and laborious approach for evaluating and resolving these equations of linear systems.

Implementation of the Kifilideen’s Extermination and Determinant of Matrix (KEDM) Method to Resolve Linear System of Three unknowns

Solve the following linear system with three unknowns using the KEDM method.

$$\begin{aligned} 7i_1 + 3i_2 + 4i_3 &= 29 \\ -8i_1 + 5i_2 - 6i_3 &= -19 \\ 2i_1 - i_2 - 9i_3 &= 47 \end{aligned}$$

Solution

Utilizing the KEDM method, we can determine the value of i_1 , with i_2 exterminated first and then i_3 , yielding:

$$\begin{vmatrix} C_{i_1} & C_{i_2} & C_{i_3} & C_{i_2} \\ 7 & 3 & 4 & 3 \\ -8 & 5 & -6 & 5 \\ -8 & 5 & -6 & 5 \\ 2 & -1 & -9 & -1 \end{vmatrix} i_1 = \begin{vmatrix} C_o & C_{i_2} & C_{i_3} & C_{i_2} \\ 29 & 3 & 4 & 3 \\ -19 & 5 & -6 & 5 \\ -19 & 5 & -6 & 5 \\ 47 & -1 & -9 & -1 \end{vmatrix} \quad (31)$$

$$\begin{vmatrix} 59 & 38 \\ -2 & 51 \end{vmatrix} i_1 = \begin{vmatrix} 202 & 38 \\ -216 & 51 \end{vmatrix} \quad (32)$$

$$3,085 i_1 = 18,510 \quad (33)$$

$$i_1 = \frac{18510}{3085} = 6 \quad (34)$$

Utilizing the KEDM method, we can determine the value of i_2 , with i_1 exterminated first and then i_3 , yielding:

$$\begin{vmatrix} C_{i_2} & C_{i_1} & C_{i_3} & C_{i_1} \\ 3 & 7 & 4 & 7 \\ 5 & -8 & -6 & -8 \\ 5 & -8 & -6 & -8 \\ -1 & 2 & -9 & 2 \end{vmatrix} i_2 = \begin{vmatrix} C_o & C_{i_1} & C_{i_3} & C_{i_1} \\ 29 & 7 & 4 & 7 \\ -19 & -8 & -6 & -8 \\ -19 & -8 & -6 & -8 \\ 47 & 2 & -9 & 2 \end{vmatrix} \quad (35)$$

$$\begin{vmatrix} -59 & 10 \\ 2 & -84 \end{vmatrix} i_2 = \begin{vmatrix} -99 & 10 \\ 338 & -84 \end{vmatrix}$$

$$4,936 i_2 = 4,936$$

$$i_2 = \frac{4,936}{4,936} = 1 \tag{36}$$

Utilizing the KEDM method, we can determine the value of i_3 , with i_1 exterminated first and then i_2 , yielding:

$$\begin{matrix} C_{i_3} & C_{i_1} & C_{i_2} & C_{i_1} \\ \left| \begin{array}{cc|cc} 4 & 7 & 3 & 7 \\ -6 & -8 & 5 & -8 \\ -6 & -8 & 5 & -8 \\ -9 & 2 & -1 & 2 \end{array} \right| \end{matrix} i_3 = \begin{matrix} C_o & C_{i_1} & C_{i_2} & C_{i_1} \\ \left| \begin{array}{cc|cc} 29 & 7 & 3 & 7 \\ -19 & -8 & 5 & -8 \\ -19 & -8 & 5 & -8 \\ 47 & 2 & -1 & 2 \end{array} \right| \end{matrix} \tag{37}$$

$$\begin{vmatrix} 10 & -59 \\ -84 & 2 \end{vmatrix} i_3 = \begin{vmatrix} -99 & -59 \\ 338 & 2 \end{vmatrix} \tag{38}$$

$$-4,936i_3 = 19,744 \tag{39}$$

$$i_3 = \frac{19,744}{-4,936} = -4 \tag{40}$$

$$i_1 = 6, i_2 = 1 \text{ and } i_3 = -4 \tag{41}$$

The Kifilideen’s Extermination and Determinant of Matrix (KEDM) Method, as demonstrated in equations (31) to (41), offers a superior approach to solving linear systems with three unknowns compared to Cramer’s rule. The KEDM method is more effective, simpler, and easier to implement than Cramer’s rule, inverse matrix method, Gaussian extermination, graphical methods, Jacobian and Gauss-Seidel methods, and LU decomposition method, due to its simplicity and efficiency. This is primarily attributed to the fact that the KEDM method only requires computing 2×2 determinants of matrices.

Compared to Cramer’s rule, the KEDM method is more efficient because it only requires computing 2×2 determinants of matrices. Consequently, the KEDM method exhibits lower complexity, convenience, and reduced computational requirements, making it a more practical and user-friendly approach. In contrast, other methods involve more complex computations. Cramer’s rule, for instance, involves more complex computations of 3×3 and 2×2 determinants of matrices. The inverse matrix method for solving three-unknown linear systems entails a multi-stage process. This process includes calculating cofactors, obtaining the transpose of the cofactors, determining the determinant, and finding the inverse of the matrix. Consequently, this procedure can be perceived as intricate, cumbersome, and prone to errors. The procedure for using the inverse matrix method is complex, nuanced, and potentially challenging to follow.

The KEDM Method also offers significant advantages over the Gaussian extermination method for resolving linear system with three unknowns. The benefits of the KEDM method include:

- A more straightforward implementation, requiring only 2×2 determinants of matrices calculations.
- Enhanced intuitiveness and ease of understanding, making it ideal for students and beginners.
- Fewer arithmetic operations, as it only involves 2×2 determinants of matrices calculations, unlike Gaussian extermination method, which requires addition, subtraction, multiplication and division.
- Reduced error propensity, as the KEDM method involves only 2×2 determinants of matrices calculations, minimizing the risk of arithmetic mistakes.

Additionally, the KEDM method provides a direct solution for each unknown, eliminating the need for back substitution, which can compromise solution accuracy. The KEDM method also avoids partial pivoting, a potential error source in Gaussian extermination method.

In contrast, the Gaussian extermination method involves multiple steps, including forward elimination, back substitution, and pivoting, making it more complex to implement and understand. This method is sensitive to floating-point arithmetic issues, such as round-off errors, particularly when dealing with ill-conditioned matrices, leading to inaccurate solutions. To mitigate numerical instability, partial or complete pivoting is required, increasing computational complexity and reducing efficiency. Ultimately, Gaussian extermination method requires a high number of arithmetic operations for 3×3 matrix making it less efficient for large systems.

The graphical methods for resolving linear systems with three unknowns, which involves visualizing three-dimensional graphs, can be challenging to interpret, making it difficult to accurately determine the solution. Moreover, the graphical method is often less accurate than the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method, as it relies on the estimating the point of intersection from the graph. Additionally, creating accurate graphs of three-dimensional surfaces can be time-consuming and error-prone. Furthermore, the graphical method is not suitable for computational applications due to its reliance on visual estimation rather than precise calculations. (KEDM) Method provides precise calculations and solutions to three-unknown linear systems.

The KEDM method also surpasses the Jacobian and Gauss-Seidel methods in terms of simplicity and efficiency. Jacobian and Gauss-Seidel methods for resolving linear systems with three unknowns rely on iterative calculations, which can be time-consuming and prone to errors. In contrast, the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method provides a direct solution for each unknown, exterminating the need for iterative calculations.

A key advantage of the KEDM method is its simplicity, as it involves only 2×2 determinants of matrices calculations. This makes it easier to implement than the Jacobian and Gauss-Seidel methods, which require matrix operations and iterative calculations. Furthermore, the KEDM method provides an exact solution in a single step, whereas the Jacobian and Gauss-Seidel methods require multiple iterations to converge to a solution. Additionally, the KEDM method does not require an initial guess for the solution, whereas the convergence of the Jacobian and Gauss-Seidel methods depends on a good initial guess. The KEDM method also allows for easy checking of the solution against the original equations and provides a finite solution. In contrast, the Jacobian and Gauss-Seidel methods can get stuck in infinite loops if the iterations do not converge. The KEDM method is also more robust than the Jacobian and Gauss-Seidel methods, as it does not require convergence tests or sensitive conditioning of the coefficient matrix. In fact, the KEDM method can be applied to systems with zero or non-dominant coefficient matrix and diagonal elements, making it a more reliable, flexible and efficient method for resolving linear systems.

Compared to the LU decomposition method, the KEDM Method offers a more efficient and intuitive approach to resolving linear systems with three unknowns. The KEDM method involves only 2×2 determinants of matrices calculations, making it simpler to implement and less prone to errors. In contrast, the LU decomposition method requires matrix factorization, back substitution, and pivoting to ensure numerical stability. This results in a higher computational cost and increased chances of arithmetic mistakes. Moreover, the LU decomposition method can introduce the round-off error, whereas the KEDM method provides an exact solution. For small systems, such as those with three unknowns, the KEDM method has a lower computational cost compared to the LU decomposition method. Additionally, the KEDM Method is more intuitive and easier to understand, especially for students and beginners, as it involves only 2×2 determinants of matrices calculations.

Overall, the KEDM method provides a more efficient, intuitive, and reliable approach to resolving a linear systems with three unknowns. Its simplicity, precision, and reduced computational cost make it an attractive alternative to traditional methods.

Utilization of the Kifilideen’s Extermination and Determinant of Matrix (KEDM) Method to Resolve Linear System of Four unknowns

The linear system with four unknowns, presented in equations (42) to (45), exhibits a system of equations with a coefficient matrix and diagonal elements that may be zero or non-dominant.

Solve this linear system using the KEDM method.

$$-4m_1 - 3m_2 + 7m_3 + 2m_4 = 5 \tag{42}$$

$$-5m_1 + 8m_3 + 6m_4 = 119 \tag{43}$$

$$-13m_1 + 9m_2 + 10m_4 = 240 \tag{44}$$

$$-m_1 + 12m_2 - 5m_3 - 3m_4 = 211 \tag{45}$$

Solution

Utilizing the KEDM method, we can determine the value of m_1 by sequentially exterminating unknowns. First, m_4 is exterminated, followed by m_3 and finally m_2 . This process yields the following:

$$\begin{matrix} C_{m_1} & C_{m_4} & C_{m_3} & C_{m_4} & C_{m_2} & C_{m_4} & C_{m_3} & C_{m_4} & C_o & C_{m_4} & C_{m_3} & C_{m_4} & C_{m_2} & C_{m_4} & C_{m_3} & C_{m_4} \\ \left| \begin{array}{c|c} -4 & 2 \\ \hline -5 & 6 \\ -5 & 6 \\ -13 & 10 \\ -5 & 6 \\ -13 & 10 \\ -13 & 10 \\ -1 & -3 \end{array} \right| & \left| \begin{array}{c|c} 7 & 2 \\ \hline 8 & 6 \\ 8 & 6 \\ 0 & 10 \\ 8 & 6 \\ 0 & 10 \\ 9 & 10 \\ 0 & 10 \end{array} \right| & \left| \begin{array}{c|c} -3 & 2 \\ \hline 0 & 6 \\ 0 & 6 \\ 9 & 10 \\ 0 & 6 \\ 9 & 10 \\ 9 & 10 \\ 12 & -3 \end{array} \right| & \left| \begin{array}{c|c} 7 & 2 \\ \hline 8 & 6 \\ 8 & 6 \\ 0 & 10 \\ 8 & 6 \\ 0 & 10 \\ 6 & 6 \\ 10 & 10 \end{array} \right| & m_1 = & \left| \begin{array}{c|c} 5 & 2 \\ \hline 119 & 6 \\ 119 & 6 \\ 240 & 10 \\ 119 & 6 \\ 240 & 10 \\ 240 & 10 \\ 211 & -3 \end{array} \right| & \left| \begin{array}{c|c} 7 & 2 \\ \hline 8 & 6 \\ 8 & 6 \\ 0 & 10 \\ 8 & 6 \\ 0 & 10 \\ 6 & 6 \\ 10 & 10 \end{array} \right| & \left| \begin{array}{c|c} -3 & 2 \\ \hline 0 & 6 \\ 0 & 6 \\ 9 & 10 \\ 0 & 6 \\ 9 & 10 \\ 9 & 10 \\ 12 & -3 \end{array} \right| & \left| \begin{array}{c|c} 2 & 7 \\ \hline 6 & 8 \\ 6 & 8 \\ 10 & 0 \\ 6 & 8 \\ 10 & 0 \\ 10 & 8 \\ -3 & -5 \end{array} \right| & \left| \begin{array}{c|c} 2 & 7 \\ \hline 6 & 8 \\ 6 & 8 \\ 10 & 0 \\ 6 & 8 \\ 10 & 0 \\ 6 & 8 \\ -3 & -5 \end{array} \right| & \left| \begin{array}{c|c} 7 & 2 \\ \hline 8 & 6 \\ 8 & 6 \\ 0 & 10 \\ 8 & 6 \\ 0 & 10 \\ 6 & 6 \\ 10 & 10 \end{array} \right| & \left| \begin{array}{c|c} 7 & 2 \\ \hline 8 & 6 \\ 8 & 6 \\ 0 & 10 \\ 8 & 6 \\ 0 & 10 \\ 6 & 6 \\ 10 & 10 \end{array} \right| \\ \left| \begin{array}{c|c} -14 & 26 \\ \hline 28 & 80 \\ 28 & 80 \\ 49 & 50 \end{array} \right| & \left| \begin{array}{c|c} -18 & 26 \\ \hline -54 & 80 \\ -54 & 80 \\ -147 & 50 \end{array} \right| & m_1 = & \left| \begin{array}{c|c} -208 & 26 \\ \hline -250 & 80 \\ -250 & 80 \\ -2830 & 50 \end{array} \right| & \left| \begin{array}{c|c} -18 & 26 \\ \hline -54 & 80 \\ -54 & 80 \\ -147 & 50 \end{array} \right| \end{matrix} \tag{46}$$

$$\left| \begin{array}{c|c} -1848 & -36 \\ \hline -2520 & 9060 \end{array} \right| m_1 = \left| \begin{array}{c|c} -10,140 & -36 \\ \hline 213,900 & 9060 \end{array} \right| \tag{47}$$

$$-16,833,600 m_1 = -84,168,000 \tag{48}$$

$$m_1 = \frac{-84,168,000}{-16,833,600} = 5 \tag{49}$$

Utilizing the KEDM method, we can determine the value of m_2 by sequentially exterminating unknowns. First, m_4 is exterminated, followed by m_1 and finally m_3 . This process yields the following:

$$\begin{array}{cccc|cccc|cccc|cccc}
 C_{m_4} & C_{m_1} & C_{m_3} & C_{m_1} & C_{m_2} & C_{m_1} & C_{m_3} & C_{m_1} & C_o & C_{m_1} & C_{m_3} & C_{m_1} & C_{m_2} & C_{m_1} & C_{m_3} & C_{m_1} \\
 \left| \begin{array}{cc|cc} 2 & -4 & 7 & -4 \\ 6 & -5 & 8 & -5 \end{array} \right| & \left| \begin{array}{cc|cc} -3 & -4 & 7 & -4 \\ 0 & -5 & 8 & -5 \end{array} \right| & \left| \begin{array}{cc|cc} 7 & -4 & 7 & -4 \\ 0 & -5 & 8 & -5 \end{array} \right| & \left| \begin{array}{cc|cc} 5 & -4 & 7 & -4 \\ 119 & -5 & 8 & -5 \end{array} \right| & \left| \begin{array}{cc|cc} -3 & -4 & 7 & -4 \\ 0 & -5 & 8 & -5 \end{array} \right| & \left| \begin{array}{cc|cc} 7 & -4 & 7 & -4 \\ 0 & -5 & 8 & -5 \end{array} \right| & \left| \begin{array}{cc|cc} 7 & -4 & 7 & -4 \\ 0 & -5 & 8 & -5 \end{array} \right| & \left| \begin{array}{cc|cc} 7 & -4 & 7 & -4 \\ 0 & -5 & 8 & -5 \end{array} \right| \\
 \left| \begin{array}{cc|cc} 6 & -5 & 8 & -5 \\ 10 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 119 & -5 & 8 & -5 \\ 240 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| \\
 \left| \begin{array}{cc|cc} 6 & -5 & 8 & -5 \\ 10 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 119 & -5 & 8 & -5 \\ 240 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| & \left| \begin{array}{cc|cc} 0 & -5 & 8 & -5 \\ 9 & -13 & 0 & -13 \end{array} \right| \\
 \left| \begin{array}{cc|cc} 10 & -13 & 0 & -13 \\ -3 & -1 & -5 & -1 \end{array} \right| & \left| \begin{array}{cc|cc} 9 & -13 & 0 & -13 \\ 12 & -1 & -5 & -1 \end{array} \right| & \left| \begin{array}{cc|cc} 9 & -13 & 0 & -13 \\ 12 & -1 & -5 & -1 \end{array} \right| & \left| \begin{array}{cc|cc} 119 & -5 & 8 & -5 \\ 240 & -13 & 0 & -13 \\ 211 & -1 & -5 & -1 \end{array} \right| & \left| \begin{array}{cc|cc} 9 & -13 & 0 & -13 \\ 12 & -1 & -5 & -1 \end{array} \right| & \left| \begin{array}{cc|cc} 9 & -13 & 0 & -13 \\ 12 & -1 & -5 & -1 \end{array} \right| & \left| \begin{array}{cc|cc} 9 & -13 & 0 & -13 \\ 12 & -1 & -5 & -1 \end{array} \right| & \left| \begin{array}{cc|cc} 9 & -13 & 0 & -13 \\ 12 & -1 & -5 & -1 \end{array} \right|
 \end{array} m_4 = \begin{array}{cccc|cccc}
 \left| \begin{array}{cc|cc} 14 & -3 & 15 & -3 \\ -28 & -104 & 45 & -104 \end{array} \right| & \left| \begin{array}{cc|cc} 15 & -3 & 45 & -104 \\ -28 & -104 & 45 & -104 \end{array} \right| & \left| \begin{array}{cc|cc} 451 & -3 & 15 & -3 \\ -347 & -104 & 45 & -104 \end{array} \right| & \left| \begin{array}{cc|cc} 15 & -3 & 45 & -104 \\ -347 & -104 & 45 & -104 \end{array} \right| \\
 \left| \begin{array}{cc|cc} -28 & -104 & 45 & -104 \\ -49 & -65 & 147 & -65 \end{array} \right| & \left| \begin{array}{cc|cc} 45 & -104 & 45 & -104 \\ 147 & -65 & 2503 & -65 \end{array} \right| & \left| \begin{array}{cc|cc} 45 & -104 & 45 & -104 \\ 147 & -65 & 2503 & -65 \end{array} \right| & \left| \begin{array}{cc|cc} 45 & -104 & 45 & -104 \\ 147 & -65 & 2503 & -65 \end{array} \right|
 \end{array} \tag{58}$$

$$\left| \begin{array}{cc|cc} -1,540 & -1,425 & -47,945 & -1,425 \\ -3,276 & 12,363 & 282,867 & 12,363 \end{array} \right| m_4 = \left| \begin{array}{cc|cc} -47,945 & -1,425 & 282,867 & 12,363 \end{array} \right| \tag{59}$$

$$-23,707,320 m_4 = -189,658,560 \tag{60}$$

$$m_4 = \frac{-189,658,560}{-23,707,320} = 8 \tag{61}$$

$$m_1 = 5, m_2 = 25, m_3 = 12 \text{ and } m_4 = 8 \tag{62}$$

The Kifilideen’s Extermination and Determinant of Matrix (KEDM) Method, presented in (46) to (62), is the most suitable methods for resolving linear systems with four unknowns. This because the KEDM method solely involves 2×2 determinants of matrices computations, making it remarkably simple and effective. In contrast, traditional methods like Cramer’s Rule, Gaussian extermination, Gauss-Seidel and Jacobian, and LU decomposition have significant limitations. Cramer’s Rule, for instance, is not ideal for resolving linear systems with more than three unknowns. Its applications to four-unknown systems are particularly challenging due to high computational complexity, which involves 4×4 , 3×3 and 2×2 determinants of matrices calculations. This complexity makes Cramer’s Rule inefficient for four-unknown linear systems and prone to implementation challenges. Furthermore, Cramer’s Rule is numerically unstable for large systems, such as those with four unknowns, leading to inaccurate solutions. The method also requires storing multiple complex determinants, resulting in substantial memory requirements. In contrast, the KEDM method does not demand extensive memory, as it entails lower-level determinant evaluations. Additionally, Cramer’s Rule necessitates calculating multiple higher-level determinants, a process that is time-consuming, error-prone, and cumbersome. In stark contrast, the KEDM method operates solely on 2×2 determinants of matrices calculations, making it more efficient and less prone to errors. In conclusion, the KEDM method surpasses Cramer’s Rule in resolving four-unknown linear systems, offering a more efficient, accurate, and simpler approach.

The graphical method has significant limitations when dealing with linear systems of more than three unknowns. As the number of variables increases, visualizing and interpreting the graph becomes increasingly challenging. Moreover, the graphical method relies on visual estimation rather than precise calculations, making it unsuitable for computational applications, particularly for four-unknown linear systems. In contrast, the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method offers a superior alternative. This method is straightforward, provides precise calculations, and yields accurate solutions, making it an ideal choice for resolving four-unknown linear systems compared to the graphical method.

Gaussian extermination is a method for resolving systems of linear equations, but it can be cumbersome when applied to four-unknown linear systems. This process involves multiple steps, including forward extermination, back substitution, and pivoting, which can make it complex to implement and understand. In contrast, the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method offers a more streamlined approach for resolving linear systems with four unknowns. By relying solely on 2×2 determinants of matrices computations, the KEDM method proves to be straightforward and effective. Furthermore, Gaussian extermination has several drawbacks. For instance, it requires a high number of arithmetic operations for 4×4 matrices, making it less efficient for large systems. Additionally, Gaussian extermination necessitates a separate step for back substitution, which can make it harder to verify the accuracy of the solution. Furthermore, careful implementation is required to handle issues of round-off errors. In contrast, the KEDM method offers a more robust and efficient approach. It allows for easy verification of the solution by simply plugging the values back into the original equations eliminating the need for separate back substitution steps. This makes the KEDM method a more reliable and efficient choice for resolving four-variable systems.

The equations presented in (42) to (45) exhibit specific characteristics. Notably, equations (43) and (44) have zero coefficients for unknowns m_2 and m_3 , respectively, which occur along the diagonal of the linear system of the equations.

To assess dominance and convergence, we evaluate the following inequalities:

$$-4 < |-3| + |7| + |2| \tag{63}$$

$$0 < |-5| + |8| + |6| \tag{64}$$

$$0 < |-13| + |9| + |10| \tag{65}$$

$$-3 < |-1| + |12| + |-5| \tag{66}$$

These inequalities indicate that the linear system of equations (42) to (45) has a non-dominant diagonal. Specifically, for each equation, the coefficient of the diagonal element is less than the sum of the coefficients of the other elements.

Consequently, the linear system of equations (42) to (45), with a coefficient matrix featuring two zero and four non-dominant diagonal elements, may not converge or may have no unique solution when using iterative methods like Gauss-Seidel and Jacobian methods. In contrast, the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method provides an exact solution for four-unknown linear systems, whereas Gauss-Seidel and Jacobian methods yield approximate solutions. The KEDM method offers several advantages, including:

- lower computational cost compared to Gauss-Seidel and Jacobian methods
- Simpler and more intuitive understanding, making it ideal for students and beginners.
- Exact solutions, as demonstrated in equation (62)

Overall, the KEDM method provides a reliable and efficient approach for resolving four-unknown systems, surpassing the limitations of iterative methods like Gauss-Seidel and Jacobian.

The Kifilideen's Extermination and Determinant of Matrix (KEDM) Method offers a simpler approach to resolving four-unknown linear systems, as it solely involves 2×2 determinants of matrices calculations. In contrast, the LU decomposition method requires matrix factorization, making it more complex to implement. Furthermore, the KEDM method requires fewer arithmetic operations, limited to 2×2 determinants of matrices calculations, whereas LU decomposition method involves both matrix factorization and back substitution. This difference in computational requirements translates to a lower computational cost for the KEDM method compared to the LU decomposition method for four-unknown linear systems. Additionally, the KEDM method is less prone to errors, reducing the likelihood of arithmetic mistakes. This increased accuracy, combined with its simplicity and efficiency, makes the KEDM method a more reliable choice for resolving four-unknown linear systems.

Application of the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method

The Kifilideen's Extermination and Determinant of Matrix (KEDM) Method is a mathematical method that illustrates the relationship between the coefficients of unknowns in linear systems of equations, the system's output, and the solution. This versatile method can be applied to resolve

linear systems with two, three, or four unknowns. A key feature of the KEDM method is its ability to demonstrate how changes in coefficients affect the solution of unknown in linear systems. By providing a predictive framework, the KEDM method enables the prediction of solutions that satisfy a linear system when the coefficients are given. This predictive capability makes the KEDM method a valuable tool for resolving linear systems of equations.

The KEDM method has a wide range of applications across various disciplines, including linear algebra, calculus, binomial system, trinomial system, tetranomial system, Kifilideen's Matrix Sequence of finite and infinite terms, physics, engineering, economics, computer, optimization, statistics, machine learning and network analysis. For example, the KEDM method can be utilized in Kifilideen's Matrix Sequence, which involves a three-variable system with infinite and finite terms. This application enables the determination of the values of the three variables: migration level value (k), migration step value (i), and first term (f) of the Kifilideen's Matrix Sequence. Furthermore, the KEDM method can be applied to find the intersection point of two lines in 2D, 3D and 4D space. The KEDM model can be applied to solve systems of linear equations in electrical engineering such as in circuit analysis. More so, the KEDM method can be used to find the maximum or minimum of a linear function in optimization problems.

Conclusion

This study makes significant contribution to the field of mathematics by successfully developing the Kifilideen's Extermination and Determinant of Matrix (KEDM) Method, a novel approach for resolving linear systems of equations with two, three, and four unknowns. Through a systematic extermination process, the method reduces the number of unknowns in a given linear system of equations in determinant forms, ultimately yielding the KEDM method. The effectiveness and simplicity of the KEDM method were demonstrated through its implementation in resolving the linear system of equations with two, three, and four unknowns. Notably, the KEDM method offers a significant advantage in resolving linear systems of equations with two, three, and four, as it solely involves 2×2 determinants of matrices calculations. This simplicity enables easier implementation, understanding and computation. The KEDM method's structured approach and coefficient arrangement enable users to efficiently apply the method to resolve a multi-unknown linear system of equations. Overall, the KEDM

method provides a reliable and efficient solution for resolving the linear systems of equations, making it a valuable contribution to the field of mathematics.

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