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Teachers' level of understanding geometry concepts vis-à-vis van Hiele's geometric thinking model

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Abstract

Understanding geometry has an impact on both the subsequent academic track and the real-life application of the learners. However, students' performance and attitude in mathematics are declining. Thus, this study examined secondary school mathematics teachers' levels of understanding of geometry based on van Hieles' geometric thinking model. The van Hiele geometry test (VHGT) result was administered to 72 teachers who joined Wolaita Sodo University in Ethiopia for the PGDT program in the summer season. The test has used 25 multiple-choice items; five items for each level of the model were adopted. A content analysis of the Ethiopian mathematics syllabus was made to check the alignment of the material with the study context. It was proven that all the concepts covered by the test items are available in the syllabus. The study revealed that teachers lack the level of understanding that is relevant to teaching geometric concepts incorporated into the secondary school curriculum. In particular, participants' mean score is 10.51 and STDEV 1.92. While the maximum score is 15, the minimum is 6, with a range of 9. While 2.8% of participant performance is below the levels, 97.2%, 59.7%, and 13.9% are able to attain the first, second, and third levels, respectively. No respondent was able to reach the fourth and fifth levels. Thus, intervention to enhance teachers' level of understanding of geometry needs immediate attention. The problem needs special attention to break the vicious circle that today's students are tomorrow's teachers, who lack the skills to scaffold their students to benefit from the education system.

Keywords: Geometric concepts, Geometric thinking level, Secondary school, van Hieles' model

Introduction

Geometry plays a good part in the Ethiopian education system. The share begins with the elementary to high school syllabus. In the university BSc program, trainees take geometry courses that aim to equip them to perform elementary geometric concepts from an advanced perspective (MOSHE, 2021). Thus, teachers' level of understanding of geometry has various implications for effectiveness in their teaching practice. A number of factors affect students learning and, hence, their effectiveness in understanding the basic concepts of geometry. The literature is full of evidence that language, visualization abilities, instructional materials, and approaches teachers use affect students' level of understanding and interest in geometry (Sunzuma et al., 2013; Knight, 2006). In the meantime, van Hieles' model of geometric thinking has been the dominant tool to explore the existing levels of understanding, identify difficulties, and prepare and implement overcoming strategies in the teaching and learning of geometry (Sunzuma et al., 2013).

The various levels of geometric thinking as described in van Hieles' model of thinking have been an influential theory in the field of mathematics education (Sunzuma et al., 2013). These levels of geometric concept formation are visualization, analysis, informal deduction, deduction, and rigor (Knight, 2006). Moreover, those authors assert that understanding geometry has an impact on both the subsequent academic track and the real-life application of the learners. Traditionally, geometry concepts have been used by some different careers: farmers when dividing plots of land, carpenters when designing house buildings, furniture makers, and merchants when measuring figures, and calculating size, and estimating cost. In modern careers too, geometric concepts have many practical applications and skills; for example, geographers make maps, architects and interior designers make blueprints, plumbers manage resources, and teachers facilitate teaching. Figure 1 illustrates some of the traditional application of geometric concepts in the Ethiopian cultural context. Even though those experts may not theoretically explain the concepts that they are implementing, their practice involves the application of complicated geometric concepts.

Figure 1. Traditional practices that are rich in geometric concepts

Students' performance and attitude in mathematics in Ethiopia are declining (Tirussew et al., 2018). In particular, teaching geometry requires more minds-on and hands-on practice from students and teachers than all other topics in mathematics. Hence, currently observed difficulties in students' performance and attitude affect more geometry teaching and learning. The literature is full of evidence about why geometry is a basic skill both for immediate real-life application and advanced learning outside the mathematics classroom (NCTM, 2000; Sherard, 1981).

In the current education policy of Ethiopia, attention is given to both informal and formal technical and vocational education and training. Vocational education has eight levels based on occupational standards. Besides, the general education curriculum framework has indicated that at the entry level of grade 11, students are required to join the vocational or academic streams, which have eight areas identified as career and technical education areas of the study (MoE, 2020). The Ethiopian secondary school teachers' preparation modality has two phases: a first degree (BSc or BA degree) qualification in the respective school subjects and PGDT (Post Graduate Diploma for Teachers) training. The first-degree program aims to equip graduates with the content knowledge necessary to teach in secondary schools. It is also aimed at equipping graduates with the required applied knowledge that can be implemented in the production and service sectors (Alemayehu et al., 2012). The same authors argue that the practical implementation has several challenges since a number of teachers did the teaching practice without taking the PGDT during their practice. Moreover, teachers face many challenges, which emanate not only from a shortage of professional courses but also from a lack of basic content knowledge itself.

In addition, the application of geometry in technical careers such as carpentry, plumbing, technical drawing, tailing, pottery, and other artifacts, nursery sites, and forestry, as well as daily life practices, demand a clear and deep understanding of geometry. Thus, having observations on difficulties in students, designing an appropriate intervention, and teachers' performance and level of understanding need to be explored. This is because teachers' level of understanding in geometry takes a good share of the observed difficulties in students (Knight, 2006; Clements, 2003).

Mathematics teachers are expected to motivate and engage students mentally, physically, and emotionally in the learning process. To do so, teachers themselves need to have a competency that makes them present concepts in simpler ways, provide real-life examples, integrate concepts with practical applications, form new examples, identify students' level of thinking, and remediate misconceptions. On the contrary, teachers themselves have difficulties and contribute to existing students' difficulties in geometry (Knight, 2006; Clements, 2003). For instance, in a study conducted in five teachers' training colleges in Ghana, Salifu et al. (2018) found that according to van Hieles' level of thinking model, only 11.44% of teachers were competent enough to implement the basic school mathematics in the syllabus. In a study conducted in the same context, Armah et al. (2017) concluded that a good number of pre-service teachers' level of thinking is lower than that of their students.

The decrease in the number of students who are joining the science and mathematics fields is a global concern. While the current interest of the Ethiopian government is to produce more science and engineering personnel, the declining number of students' performance and lack of interest is a national concern (United Nations, 2019; Tirussew et al., 2018; Bethell, 2016). On the other hand, research in mathematics education in general and investigation in geometry in particular in Ethiopia is at its infant stage since there are only a few investigations in geometry in the context of Ethiopia.

The study was aimed at investigating secondary school teachers' level of understanding geometric concepts based on van Hieles' geometric thinking model. The study was guided by the following research question: 1/ What is the teachers' level of understanding geometric concepts as measured by van Hieles' level of geometric thinking?, and 2/ How is the pattern of teachers' level of understanding with regard to the five levels of van Hieles' geometric thinking model?

The findings of this study are expected to reveal teachers' level of understanding of geometric concepts and, hence, the implications for students learning. Basically, it will serve as a baseline for determining who and when to prepare an intervention. Besides, the findings will have implications for the width and breadth of the required intervention to enhance students' learning. One mandate of universities in Ethiopia is to provide need-based community service to the catchment area community. The finding will add value to the university's mission of improving the quality of education in the catchment area, inform regional education departments and the Ministry of Education about the existing practice and the impact of the current teacher training modality, and contribute to the literature in the field.

Theoretical framework

The Van Hieles' geometric thinking model

The Dutch researchers and mathematics teachers, Pierre van Hiele and his wife Dina van Hiele-Geldof, found the initial idea of the model based on their own interactions with the difficulties that their students encountered around the late 1960s. With different educators' and researchers' contributions and modifications, the model currently suggests that geometric thinking and one's concept formation have five sequential and hierarchical levels. The five levels in their ascending order of difficulty are recognition (visualization), analysis, order (informal deduction), deduction, and rigor (Knight, 2006).

While different researchers gave descriptions of the levels (Senk, 1989; Burger and Shaugnhessy, 1986), the description by Karakus and Peker (2015) is more explicit. Table 1 presents a description of the levels, the mental structure of an individual at a given level, and an example for the level demonstration based on Karakus and Peker (2015).

Teachers' understanding level of geometry, according to van Hiele's model, is significant for their teaching practice. Supporting this idea, Knight (2006) mentioned that ‘‘teachers must present the material within this level to enable the student to master the content at this level and move on to the next level''. In general, the model is dominant in the study of geometry as it best estimates learning and hence demonstrates what is learned (Atebe, 2008; Knight, 2006) and was used to underpin this study.

Table 1. Description of the five levels of van Hieles' geometric thinking

Levels	Description	Required mental structure	Example of demonstration / profile
I Visual	• Recognize figures by their appearance. • Identify a shape in specific orientation but not specify individual parts or properties	They make decisions based on intuition, not reason	They name any four side figure as a rectangle whether it is a rectangle or square
II Analysis	• Recognize figures by their properties and hence start to create classes of figures • But still lack to explain the relationship between different properties	Make decisions based on memorization.	Recognize that all Quadrilaterals have four sides and all triangles have three angles.
III Ordering	• Start establishing an interrelationship between properties, either within a class of figures or among a class of figures.	Reproduce a proof when starting from a different or unfamiliar premise.	Understand that a square is a special case of rectangles because it has all the properties of a rectangle
IV Deduction	• Construct theorems within an axiomatic system. • Understand and see the role of undefined terms, postulates, definitions, theorems, and proofs.	Understand the difference between necessary and sufficient information	Explain common and different properties of a quadrilateral to be square or a rhombus
V Rigor	• Understand the relationship between various systems of geometry. • Compare, analyse, and create proofs under different geometric systems.	Transfer understanding and compare different axiomatic systems	Proof the area of a triangle is $A_T = \frac{1}{2}bh$ from area formula $A_R = bh$ of a rectangle

Common properties through the development of the levels

According to Knight (2006), the five levels of geometric thinking in van Hieles' model have additional common properties. These properties are identified as fixed sequence, adjacency, distinction, separation, and attainment.

i. Fixed sequence is explained as the failure of the student to pass or progress to the $n+1$ levels of understanding without first having attained a level n where $n = 1,2,3,4$.
ii. Adjacency is defined as the ability to recognize that the properties of an object, which are intrinsic at one level, are extrinsic at the next.
iii. Distinction is described as the ability to use and understand the vocabulary associated with the level.
iv. Separation is defined as the inability of two people who are at different levels to understand each other.
v. Attainment or advancement outlines the learning process that leads to complete understanding at the next higher level.

As mentioned above, the various levels of geometric thinking as described in van Hieles' model of thinking have been an influential theory in the field of mathematics education. The literature has empirical evidence about measuring teachers' level of understanding geometry concepts using the model. For instance, in a study conducted in five teacher training colleges in Ghana, Salifu et al. (2018) found that according to van Hieles' level of thinking model, only 11.44% of teachers were competent enough to implement the basic school mathematics in the syllabus. Besides, the findings revealed that while 37.2% have not reached any of the levels, 62.7%, 30.2%, 9.4%, and 1.4% of the participants have reached levels 1, 2, 3, and 4, respectively. Thus, their study findings have a better result in terms of attaining the 4th level, whereas our study has a better result in terms of attaining levels 1 to 3.

A study conducted at schooldays schools in Nigeria and South Africa concluded that most grade 10 to 12 students are not ready enough for formal deductive study of geometry (Atebe, 2008). While the study found that collectively 2% and 3% of participants reached levels III and IV of van Hieles' level of geometric thinking, the South Africans performed better than their Nigerian counterparts of the sampled participants. Besides, the study revealed that scores on the van Hiele's geometric test (VHGT) strongly correlate with performance in any other geometry content score and performance in mathematics. On the same level of students studied in Indonesia, Naufal et al. (2020) also found that almost all the respondents failed to reach the ordering (informal deduction) and deduction levels, i.e., level III and IV of the van Hieles' thinking level.

Methodology

The study employed a survey design to investigate teachers' level of understanding of geometry in comparison to the van Hieles thinking model. The study population was 72 teachers who joined Wolaita Sodo University for the PGDT program in the 2020 summer season. All volunteered mathematics trainees at entry-level were selected to participate in the study, and the sampling technique is availability sampling.

Data collection instrument

For the data collection, the van Hiele geometry test (VHGT) of 25 multiple-choice items, five items for each level, was adopted (Usiskin, 1982). A content analysis of the Ethiopian mathematics syllabus was made to check the alignment of the material with the study context. It was proven that all the concepts covered by the test items are available in the syllabus. Figure 2 is a sample of the items from levels 2 and 3, respectively.

Figure 2. Sample items from the test

PQRS is a square.

Which relationship is true in all squares?

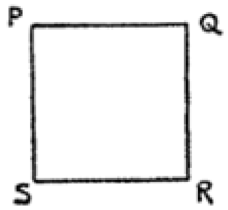
(A) $\overline{PR}$ and $\overline{RS}$ have the same length.

(B) $\overline{QS}$ and $\overline{PR}$ are perpendicular.

(C) $\overline{PS}$ and $\overline{QR}$ are perpendicular.

(D) $\overline{PS}$ and $\overline{QS}$ have the same length.

(E) Angle Q is larger than angle R.



Which is true?

(A) All properties of rectangles are properties of all squares.

(B) All properties of squares are properties of all rectangles.

(C) All properties of rectangles are properties of all parallelograms.

(D) All properties of squares are properties of all parallelograms.

(E) None of (A)-(D) is true.

Data analysis

The scoring criteria were also adopted from Usiskin (1982). Accordingly,

i. If a participant answers three or more first-level questions correctly, it is considered that he/she has reached the first level.
ii. If a participant reached the first level, answered three or more items correctly from the second level, but failed to correctly answer three or more questions from subsequent levels (levels 3, 4, and 5), s/he was classified in the second level.
iii. Using the same criteria set by Usiskin (1982), the passing rate of this study was set at 60%. If the scores of the participants did not follow the criteria, the cases were labelled "jump phenomenon" by the authors.

Equally 1 mark for each correct answer was allocated for the test items. Thus, the marking schema lies between 0 and 25. As recommended by Usiskin (1982) and also used by Salifu et al. (2018) for each block of five items, '3 out of the 5' was used as the correct success criterion for the level assignment. The questions are arranged sequentially, in blocks of 5 questions each, in ascending order of difficulty.

Results

The results of the 72 participants for each item in the levels are presented in Table 2. The data revealed that all participants attempted every item. The analysis of the performance on items in each level is presented next to the table. The highest level was an informal deduction, achieved by a group of 10 participants. In contrast, the statistics showed that no one reached the fourth level of formal deduction.

Table 2. Test results of the 72 participants

No.	L1	L2	L3	L4	L5	Total	No.	L1	L2	L3	L4	L5	Total	No.	L1	L2	L3	L4	L5	Total
1	5	4	2	1	3	15	25	5	4	2	0	0	11	49	3	2	1	2	1	9
2	5	4	4	0	1	14	26	4	4	2	1	0	11	50	3	3	1	1	1	9
3	5	4	2	1	2	14	27	3	5	1	1	1	11	51	4	3	0	1	1	9
4	4	4	3	2	1	14	28	4	4	2	0	1	11	52	4	2	1	1	1	9
5	4	4	2	1	2	13	29	4	4	1	1	1	11	53	3	3	2	0	1	9
6	4	4	4	0	1	13	30	5	3	2	1	0	11	54	4	2	1	1	1	9
7	5	4	2	2	0	13	31	4	3	3	0	1	11	55	4	1	1	1	2	9
8	3	4	4	1	1	13	32	3	3	2	1	2	11	56	3	1	1	3	1	9
9	5	3	4	1	0	13	33	5	3	1	2	0	11	57	4	0	3	2	0	9
10	5	4	2	1	1	13	34	4	2	3	0	2	11	58	4	1	1	1	2	9
11	5	5	2	1	0	13	35	4	3	2	1	1	11	59	5	2	1	1	0	9
12	4	4	0	1	3	12	36	5	3	0	1	2	11	60	3	2	2	0	2	9
13	3	4	3	1	1	12	37	5	1	1	2	1	10	61	4	0	0	3	2	9
14	4	3	3	0	2	12	38	3	4	1	2	0	10	62	4	3	2	0	0	9
15	5	4	1	2	0	12	39	4	3	2	0	1	10	63	2	1	2	2	1	8
16	3	4	3	2	0	12	40	4	2	3	1	0	10	64	5	1	1	1	0	8
17	4	4	1	0	3	12	41	4	3	2	0	1	10	65	4	1	1	2	0	8
18	5	5	2	0	0	12	42	4	2	3	1	0	10	66	2	3	2	0	0	8
19	4	3	2	1	2	12	43	5	3	1	1	0	10	67	5	1	0	1	1	8
20	4	3	3	2	0	12	44	4	3	1	1	1	10	68	4	2	0	1	1	8
21	4	2	2	2	2	12	45	4	3	1	0	2	10	69	4	1	1	0	1	7
22	5	2	3	1	1	12	46	4	1	1	2	2	10	70	3	2	2	0	0	7
23	5	4	2	0	1	12	47	4	2	4	0	0	10	71	3	2	1	1	0	7
24	4	5	2	1	0	12	48	5	1	1	3	0	10	72	3	2	0	1	0	6
Mean															4.05	2.79	1.76	1	0.90	10.51
Standard Deviation															0.77	1.24	1.05	0.80	0.85	1.92

Note: L = Level

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Performance on items in each level

In the items of the first level, respondents performed well, with a mean score of 4.05 and STDEV of 0.77. Among them, 70 (97.2%) got 3, 4, and 5, thus reaching the first level and having opportunities to go beyond. Only two respondents (respondents under numbers 63 and 66) failed to score 3 or more in the first level and were hence considered below the level. In the items of the second level, respondents again performed well, with a mean score of 2.79 and a STDEV of 1.24. Among the 70 (97.2%) participants who got 3, 4, and 5 in the first level (i.e., had attained the first or visualization level), 27 (37.5%) failed to get 3 or more in level two, and hence only 43 (59.7%) have reached the second level (i.e., had attained the second level or analysis level). In addition, among these 43 (63.8%) who reached the second level, only 10 (13.9%) got 3, 4, and 5 in the third level (i.e., had attained the third or ordering level). Furthermore, among the 10 (13.9%) who passed to the third level, all of them scored less than three in the fourth level. Hence, no one reached the fourth or fifth levels. While Table 3 presents the description of respondents' scores (count and percentage) on each level, Table 4 presents the pattern of levels of attainment of van Hieles' geometric thinking of respondents as compiled from Table 1.

Table 2. Description of respondents' scores in each level

Score (out of five)		5	4	3	2	1	0	level
	N	21	35	14	2	0	0	I
	%	29.1	48.6	19.5	2.8	0	0	
	N	4	20	20	15	11	2	II
	%	5.5	27.8	27.8	20.8	15.2	2.8	
Frequency	N	0	5	11	25	24	7	III
	%	0	6.9	15.2	34.7	33.3	9.7	
	N	0	0	3	14	35	20	IV
	%	0	0	4.1	19.5	48.6	27.8	
	N	0	0	3	14	28	27	V
	%	0	0	4.1	19.5	38.9	37.5	
		25	60	51	70	98	56	360 = 72 x 5

The data in these tables shows how the performance of participants in the current study area relates to the theoretical model presented in Table 1. Figure 3 is also an alternative representation of the data. Table 2 shows the score pattern of participants in the test items.

Table 3. Pattern of attained levels

	Respondent per attained level					
	V	IV	III	II	I	Below level
Number	0	0	10	43	70	2
Percentage	0	0	13.9	59.7	97.2	2.8

Overall performance on the test

According to the data in Table 2, participants' mean score and STDEV were 10.51 and 1.92, respectively; while the maximum, minimum scores and range were 15, 6, and 9, respectively (Figure 3).

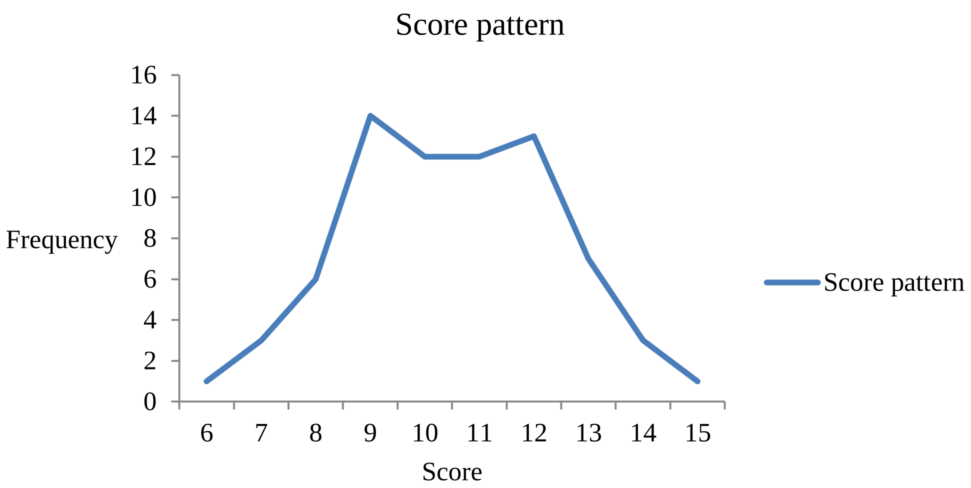

Figure 3. Score pattern of participants

Discussion

It has been found that, except for two participants, all the others had attained a visualization level of geometric thinking. The highest level was an informal deduction, which was achieved by 10 participants. On the contrary, the data revealed that no one reached the fourth level of formal deduction, let alone passed it. The result is disgusting as compared to the demands of the syllabus objectives and the expectations from the teachers to attain those intended objectives. For instance, the chapter objectives in the Ethiopian grade 9 textbook constitute the following statements: (i) know important properties of regular polygons and use the properties to solve related problems; (ii) use postulates and theorems on congruent and similar figures and solve related real-life problems; (iii) solve real-life problems on distance, and angle using their knowledge and skills in trigonometry; (iv) use symmetrical and angle properties of circles to solve related problems; (v) calculate arc lengths, perimeters, and areas of segments and sectors; calculate areas of triangular and parallelogram regions; and (vi) calculate surface areas and volumes of cylinders and prisons. To achieve those objectives, making, testing, and proving conjectures at level 4 of van Heile's thinking model is required. Hence, for effective teaching, teachers' preparation should be beyond this level. Supporting this, NCTM (2014) states that secondary school students were supposed to reach out to level 3, which is formal deduction, before entering the university or college.

Thus, it is reasonable to conclude that teachers need additional content knowledge in geometry for secondary mathematics teaching (Knight, 2006). This result has some similarities and differences with findings in the literature in terms of the percentage of those who reached each level (Naufal et al., 2020; Salifu et al., 2018; Atebe, 2008).

With this level and pattern of teacher understanding, it is not expected to attain the syllabus objectives. Besides, less performance in geometry has many negative effects on achieving expected performance in mathematics, as it is vital for understanding advanced concepts through capacitating the preconception and development of attitude (Sunzuma et al., 2013). The literature is evidenced by interventions that enhanced teachers' performance and understanding (Armah and Kissi, 2019). Naufal et al. (2020) state that the traditional teaching and learning approach has failed to increase the student's van Hiele level of geometric thinking. Thus, in order to achieve each van Hiele level of geometric thinking, students should go through phase-based instruction, i.e., go through all five learning phases, to advance from the basic level to the next stages (Meng, 2009).

Conclusion

This study examined secondary school mathematics teachers' level of understanding geometry based on van Hieles' thinking model. The result disclosed that the current teachers' training practice has drawbacks in preparing teachers for van Hieles' geometric thinking model. Hence, a lot has to be done to enhance teachers' level of understanding. The study has potential in terms of informing researchers and educators in terms of preparing interventions to overcome observed difficulties and, hence, redesigning teachers' geometric thinking. The suggestion of creating a teaching and learning model adjusted to each level of geometric thinking of students seems more relevant and demands attention. Intervention to enhance teachers' level of understanding needs immediate attention. The problem needs special attention to break the vicious circle that today's students are tomorrow's teachers, who lack the skills to scaffold their students' level of understanding. Further studies have to be conducted with a larger sample size and in different settings. In addition, further intervention studies that address how to enhance teachers' level of understanding and hence do well to best benefit their students need attention.

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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